Visualization in Learning Mathematics with Hypervideo

Teresa Chambel, Lara Santos
Dep. de Informática, Faculdade de Ciências, Universidade de Lisboa
Edifício C6, Piso 3, Campo Grande, 1749-016 Lisboa, Portugal
tc@di.fc.ul.pt, lara.santos@netcabo.pt

Suzana Nápoles
Faculdade de Ciências da Universidade de Lisboa, CMAF
Avenida Prof. Gama Pinto, 2, 1649-003 Lisboa, Portugal
napoles@ptmat.fc.ul.pt

José Francisco Rodrigues
Dep. de Matemática, Faculdade de Ciências, Universidade de Lisboa
Edifício C6, Piso 2, Campo Grande, 1749-016 Lisboa, Portugal
rodrigue@ptmat.fc.ul.pt

Tom Apostol
Caltech - California Institute of Technology
253-37 Caltech, Pasadena, CA 91125, USA
apostol@caltech.edu

ABSTRACT

Visualization has always been an essential aid in the communication of mathematics. It is an important way to concretize concepts, to develop abstraction skills, and to motivate learning, for example in topology and geometry, and in the application of numerical methods to simulations of the real world. Video has proven to be one of the most adequate ways to communicate visualization results, allowing to present in a rich cultural context a large quantity and diversity of information in a brief period of time. However, by itself, video has a limited capability to support learning. The structure and interaction introduced by hypervideo allow providing the user with greater control and autonomy, exploring links among the information conveyed by the video and complemented by other materials, augmenting its capabilities as a cognitive artifact. This paper develops these ideas, presenting The Story of Pi hypervideo as a case study.

INTRODUCTION

The visual representation of mathematical ideas, principles or problems has always played an important role in both teaching and learning mathematics. Computer graphics tools have been fundamental in visualization, and video is one of the best ways to communicate visualization results. The combination of computer animation and video is particularly powerful to capture the viewer's attention and appeal to her intuition, showing that learning mathematics can be exciting and intellectually rewarding. Video makes possible to transmit a large amount of information in a short time, not being expected that all the information will be understood and absorbed in a single viewing. The structure and interaction introduced by hypervideo allow the user greater control and autonomy. The following sections discuss how visualization can be used to support learning mathematics, how video can be used as an effective vehicle in this domain and how the structure and interaction in hypervideo augment their capabilities as cognitive artifacts, by exploring links among the information conveyed by the video and complemented by other materials. As a case study, The Story of Pi video and hypervideo are presented.

VISUALIZATION IN LEARNING MATHEMATICS

Abstraction is essential in mathematics, allowing the generalization of ideas and results, leading to advances that go farther than our conception of the real world. It has also made mathematics a difficult subject to learn, especially for those students who need to see, touch or try what they are learning [8]. Concretization makes the inverse way from abstraction to concrete concepts, and plays a key role in learning. Visualization is an important way to concretize mathematical concepts, and may represent mathematical objects that do not have a real existence. It has been playing an important role in topology and geometry and in the application of numerical methods in simulations of the real world. The familiar applications of mathematical visualization techniques are related with the representation of mathematical surfaces that were usually illustrated by plaster models and drawings, as in [10]. The advantage of supplementing this type of representations by computer generated images goes beyond the underlying simplicity and speed: it then becomes straightforward to create animations that can bring the known mathematical landscape to life in unprecedented ways. According to Hoffman [11], the computer-created model is not restricted to the role of illustrating the end product of mathematical understanding, as the plaster models are: they can be part of the process of doing mathematics. The use of mathematical visualization software makes possible to obtain fresh insights concerning complex and poorly understood mathematical objects. It can review hidden properties that were not apparent from the theoretical description [18]. Applied mathematicians find that the interactive nature of the images produced allows them to do mathematical experiments with an ease never before possible, and scientists who need and use mathematics can often better understand the mathematical concepts they have to deal with using visual embodiments. Finally, no one denies the aesthetical aspects of visualization.

LEARNING WITH VIDEO

Video, as dynamic and figurative information combined with verbal audio, forms a powerful means of communicating meaningful-scenarios rapidly and efficiently [12,17]. It can bring context to topics and enhance the authenticity of a computer based learning environment. In some learning situations, videos or animations are not only a desirable, but an important prerequisite for successful learning to take place. From a cognitive perspective, they can support learning: by *'replacing' real experience*, because of their authenticity and realism; by *visualizing dynamic processes*, which might not be observable in reality or which are hard to describe verbally; by *combining diverse symbol systems*, such as pictures, texts and narration, into coherent media messages; and through the *conducting of video projects*, where learners engage in active video production [7,14,19].

To show that mathematics is understandable, exciting, worthwhile, and intellectually rewarding, video has been used in high-quality instructional modules. Some videos are eminently suitable for school or university instruction, others target a more general scientific audience potentially interested in mathematical topics and their visualization. The mathematical themes addressed include problems in topology and geometry and their recent solutions, visualizations of classical ideas of mathematicians like Archimedes, Eratosthenes, Pythagoras, and Fibonacci, history of mathematics, topics in high school mathematics, and applications of modern numerical methods to real world simulations. Video treats mathematical concepts in ways that cannot be done at the chalkboard or in a textbook. They use live action, music, special effects, and imaginative computer animation, and can transmit a large amount of information in a relatively short time. Consequently, it is not expected that all students will understand and absorb all the information in one viewing. The viewer is encouraged to take advantage of video technology that makes it possible to stop it and repeat portions as needed. In another hand, to allow reflection, a system must have a medium that affords adding, modifying and manipulating representations, and performing comparisons. It must also afford time for reflection, elaboration, and comparison processes. Broadcast television, and most videos, are usually watched in an experiential mode, and cannot augment human reflection in this sense [16].

HYPERVIDEO AS A COGNITIVE ARTIFACT

Hypervideo refers to the true integration of video in hypermedia spaces, where it is not regarded as a mere illustration, but can also be structured through links defined by spatial and temporal dimensions [3,5,6]. It greatly augments video capabilities, by providing flexible interactive mechanisms that integrate and navigate different types of media, in ways that can be adapted to a great variety of learning styles and situations [6,8]. We have been exploring and developing different technological and methodological approaches for the design and construction of hypervideo, with a special concern to the support of learning processes [3,4,5,6,8]. From our studies and experiences, we concluded that hypervideo provides a more flexible and engaging way of watching videos, increasing students' motivation to watch them. By providing different indexes to the video, it is easier to search and access information and to capture the videos' messages. Important relations between video and other media, like text, through contextualized explanations and illustrations can be captured by this full integration of video in hypermedia, promoting deeper understandings of the underlying materials. Different learning styles are also supported, through the integration of various media, perceptual modalities, and interactive navigational choices [5,6,8].

THE STORY OF PI – VIDEO

The Story of Pi video discusses the early history of Pi and shows how it appears in a variety of formulas and applications, many of which have nothing to do with circles, namely in probability problems. It is part of a series of modules, developed under Project Mathematics! [20], to introduce basic mathematical concepts in high school or community college, using live action, music, special effects, and imaginative computer animation that bring mathematics to life. The animation, together with images of historical documents and applications to the real world, grabs the attention of students and motivates them to want to learn more [1,2]. The video describes a sequence of improved estimates for the value of Pi, points out that Pi is irrational and explains that major improvements in the estimates for Pi represent landmarks of important advances in the history of mathematics. It uses different visualization techniques to present and illustrate various properties and applications of Pi, as exemplified bellow.

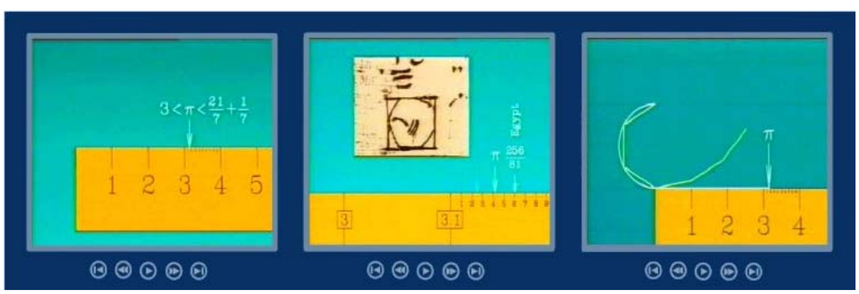

Figure 1 – Sequence of early attempts to estimate the value of Pi.

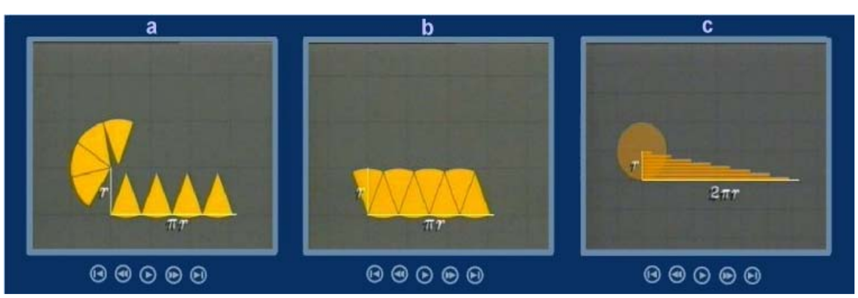

Figure 2 – Animated derivations of the formula for the area of a circle.

Figure 1 exemplifies the use of animations based on a ruler to show the relative positions of the estimates of Pi obtained in early times, providing an opportunity for discussing the practical difficulties of making accurate measurements of real objects. Two and three dimensional animations are used to provide detailed explanations of formulas involving Pi. In figure 2a,b, to explain why the area of a circle with radius **r** is **Pi*r**^{2}, the circle is divided in a large number of radial slices, and the animation shows how to rearrange the slices to form a new figure that is almost a parallelogram with the same area as the disk. In figure 2c, the disk is divided into equally spaced concentric rings that are unwrapped and stacked into a pile that looks somewhat like a right triangle. The base of the triangle is nearly the circumference of the circle and its altitude is equal to the radius. So, once more we see why the area of the circle disk is **Pi*r**^{2}. To explain the surface areas and volumes of revolution solids, the animation moves two-dimensional shapes to become three-dimensional objects. Figure 3 illustrates the surface area and volume of a revolution cylinder. Using a similar approach, other examples in the video address cones, and torus.

Figure 3 – Animated derivations of the formulas for the surface area and volume of a revolution cylinder.

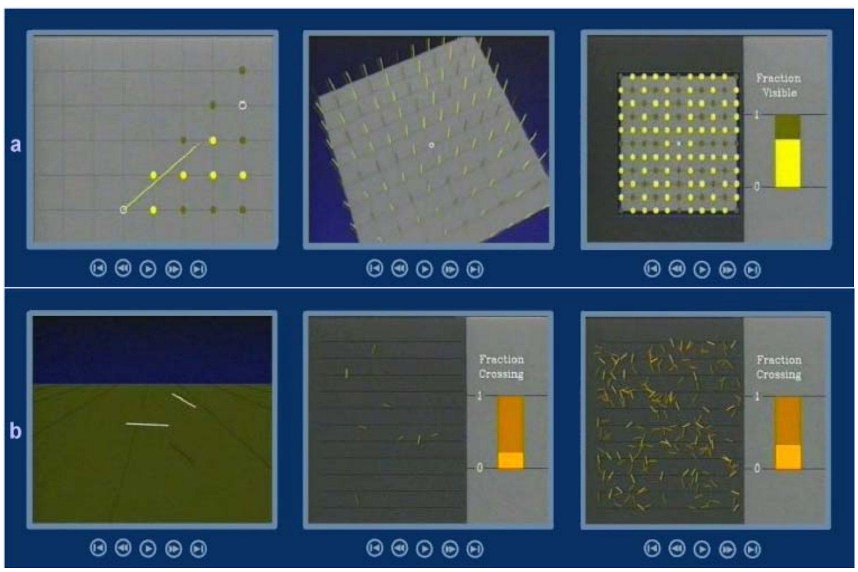

Figure 4 – a) Animation showing lattice points visible from the origin; b) The Buffon needle problem.

The animations in figure 4 provide an explanation for how Pi appears in some random events. Using two and three dimensions, the first sequence describes visibility of lattice points from a fixed origin. The computer animation reveals that about **2/3** of the lattice points in the plane are visible from the origin, with exact probability being **6/Pi**^{2} that a lattice point chosen at random will be visible from the origin. The second sequence effectively illustrates with animation the Buffon needle problem. A large number of needles of the same length are shown falling at random on a pattern of equally spaced parallel grid lines, whose spacing is twice the length of the needles, and the exact probability that a needle crosses a grid line is **1/Pi**.

The video also captures real images to illustrate experiences involving Pi, providing connections with real life and more authentic situations. Figure 5a corresponds to a sequence presenting an experiment concerning the definition of Pi as the constant ratio between the circumference and diameter of a circle. In figure 5b, a shot of the earth is used to show the great circle route of an airliner flying from New York to Tokyo. The animation was timed for the approximate duration of flight relative to the rotation of the earth. The third image (figure 5c) illustrates an ancient circular monument, for which construction Pi had to be considered.

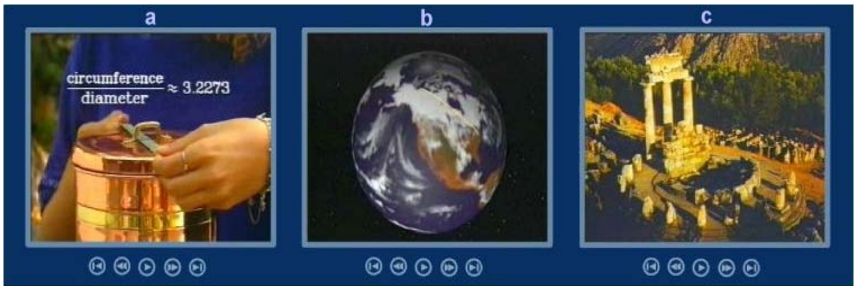

Figure 5 – a) Measuring the circumference of circles and correspondent diameters in real life objects; b) An airliner flying from New York to Tokyo; c) Pi in ancient architecture.

The Story of Pi video is distributed with a workbook that elaborates on the important ideas in the video and provides exercises for the students. For the current application, the main ideas of the workbook were included in an extended textbook that provides more advanced information regarding topics related with Pi, enriching its content as a support for learning in either high school or university levels.

THE STORY OF PI – HYPERVIDEO

The hypervideo developed for The Story of Pi structures and integrates the video with other materials, augmenting their individual affordances to support learning. Video can be navigated from different types of indexes that are presented in synchrony with the video. The textbook was converted into hypertext, respecting its underlying hierarchical structure, and further enriched with links involving text, images, video, and applications, allowing to capture relations among them and to illustrate or complement the information conveyed by each one. Some features of the hypervideo are presented in the following examples.

Figure 6 illustrates the different types of indexes or maps, on the right, provided for the video, on the left. The first index presented (figure 6a,b) shows an overview of the video chapters through a title and a representative image for each one of them. Figure 6d exemplifies an image-based index, containing an image for each relevant scene in the video. It allows for a visual summary and search in the video. The index presented in figure 6c is a text-based index containing the table of contents of the video, its chapter, sections and sub-sections. It allows for a semantic summary and search of the video.

The user can choose and switch among the three indexes, as exemplified. All of them are synchronized with the video, highlighting the current topic, with a white frame or text color, contrasting with the rest. It is also possible to enter the video in any of the indexes' entry, providing the user with great control over the presentation, in a contextualized fashion. Beneath the video, some controls also allow the traditional VCR navigational capabilities of play, pause, forward and rewind. These can be done by short leaps or by chapter. A timeline permits random access and indicates the current position in the video.

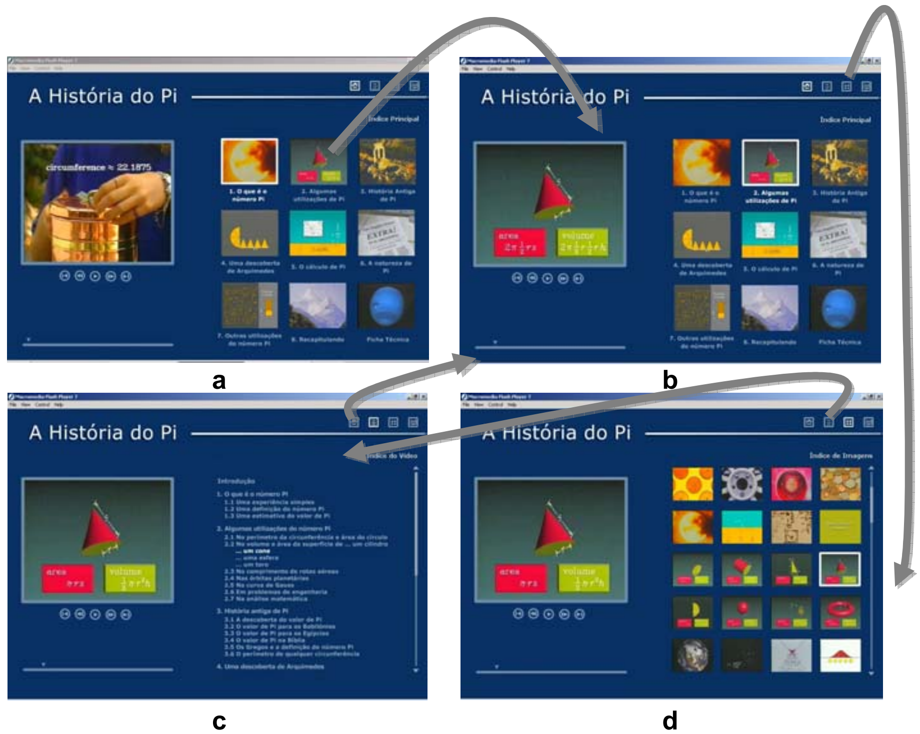

Figure 6 – Video indexes in The Story of Pi hypervideo:
a) b) overview index; c) text index; d) image index.

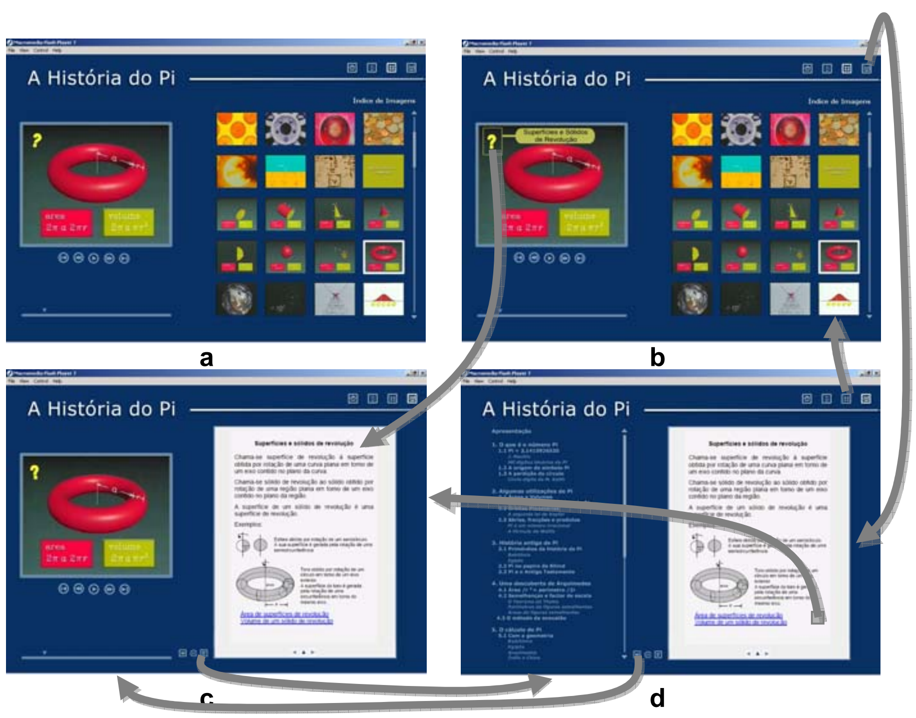

Figure 7 – Navigation in the hypervideo, integrating the video and the textbook.

The example presented in figure 7 illustrates the integration and navigation of the video and the textbook. While watching the explanation about the toru's volume, a question mark pops up on the video to indicate a link (figure 7a). While the cursor is over the question mark, a tag appears informing the user about extra information on this topic. If the user clicks on the mark, the textbook is opened on the page providing additional information on the topic (figure 7b→c). The user may decide to continue watching or listening to the video, while reading the page, or she may decide to pause the video. It is also possible to access the textbook index, also synchronized with the page opened (figure 7c→d). The user can switch between the view of the video and the view of the book index at any time, while reading the text (figure 7c↔d). Also note that the book can always be accessed directly through the index button (figure 7b→d). At some points in the text, it is also possible to access the video for an additional explanation or an illustration of what is being read (figure 7 upper arrow in d→c). This way, the information is complemented and contextualized in the different media.

In more restricted situations, where the video segments presented are not included in the main video, images on the textbook may come to life to provide the additional and contextualized information. This is the case presented in figure 8a, where the video segment illustrates the scale factor effect concerning perimeter and areas in similar shapes. Interactive applications can also be integrated to illustrate and explore some aspects, allowing for a better understanding of the topics. Figure 8b presents one such application for the Keppler's 2nd Law, developed in Cinderella [9].

a b

Figure 8 – a) Video segment embedded in the textbook; b) Integration of the Keppler's 2nd Law application.

This hypervideo was developed in Macromedia Flash [13]. Previous prototypes were developed in HTIMEL [3,4] and SMIL [15,21,22]. There are pros and cons in each one for the development of this type of hypervideo, in terms of specific features, programming paradigms, flexibility, availability and portability, among others, but this discussion is beyond the scope of this paper.

CONCLUDING REMARKS

Visualization has always played an important role in both teaching and learning mathematics. Video provides a highly efficient way of conveying audiovisual information. If one picture is worth a thousand words, a 20-minute video must be worth million of words. Visual images make a much greater impact than printed or spoken words alone, but video by itself is insufficient to support learning as it provides a large amount of unstructured information in a short time. The interaction introduced by hypervideo, in exploring links established within and between video and other materials, provides great flexibility, control, autonomy and motivation to the viewer. By providing different indexes to the video, it is easier to search for information and to capture the videos' messages. Important relations between video and other media, like text, through contextualized explanations and illustrations can be captured by this full integration of video in hypermedia, promoting deeper understandings of the different materials. Through the integration of various media, perceptual modalities, and interactive navigational choices, hypervideo may be used for different levels of knowledge and learning styles. Hypervideo can thus provide a valuable pedagogical support to reveal mathematics for what it is, not only understandable and exciting, but eminently worthwhile as well.

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